

Arithmetic Mean (Average)

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Add all values, then divide by the number of values.

When to use: Normal distributions, no outliers, all values matter equally

Example: $(2 + 4 + 6) \div 3 = 4$

Geometric Mean

$$G = \sqrt[n]{x_1 \times x_2 \times \dots \times x_n}$$

Multiply all values, then take the n-th root.

When to use: Growth rates, percentages, ratios, positive values only

Example: $\sqrt[3]{2 \times 4 \times 8} = \sqrt[3]{64} = 4$

Harmonic Mean

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

Divide number of values by sum of reciprocals.

When to use: Rates, speeds, frequencies, time-based calculations

Example: $3 \div (1/2 + 1/4 + 1/8) = 3 \div 0.875 \approx 3.43$

Weighted Mean

$$\bar{x}_w = \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + \dots + w_n}$$

Multiply each value by its weight, sum, then divide by total weight.

When to use: Different values have different importance or frequency

Example: $(2 \times 1 + 4 \times 2 + 6 \times 1) \div (1 + 2 + 1) = 16 \div 4 = 4$

Median (Middle Value)

Odd n: Middle value when sorted

Even n: $M = \frac{x_{n/2} + x_{(n/2)+1}}{2}$

Sort values first. If odd count: take middle value. If even count: average the two middle values.

When to use: Skewed data, presence of outliers, ordinal data

Examples: $[2, 4, 6] \rightarrow \text{median} = 4$ | $[2, 4, 6, 8] \rightarrow \text{median} = (4 + 6) \div 2 = 5$

Mode (Most Frequent)

The value(s) that appear most frequently in the dataset

Count frequency of each value. The value with highest frequency is the mode.

When to use: Categorical data, finding most popular item, nominal data

Examples: $[2, 4, 4, 6] \rightarrow \text{mode} = 4$ | $[1, 2, 2, 3, 3] \rightarrow \text{modes} = 2 \text{ and } 3$ (bimodal)